Stability of a Dual-Spin Satellite with Two Dampers

F. R. VIGNERON*
National Space Telecommunications Laboratory, Communications Research Centre, Ottawa, Canada

Linearized equations of motion for a dual spin satellite composed of a platform, a rotor, a platform-mounted damper, and a rotor mounted damper are solved by the method of averaging to obtain a closed-form first approximation solution. The results obtained are found to be in good agreement with numerically computed solutions. Analytical stability criteria are also obtained and verified by comparison with the results of Floquet analysis. The method offered in the paper is based on the established formalism of the method of averaging, and in principle will yield a complete solution to any degree of approximation desired. In this application, the solution is noted to be particularily accurate when the masses of the dampers are small with respect to the total satellite mass, which is the case in current satellite designs.

Nomenclature

a = distance from pendulum damper mass to 0
A_0 = moment of inertia of undeformed configuration about $0'x'$ or $0'y'$
$A(\tau),B(\tau)$ = variables of motion defined in Eq. (13)
a_{11},a_{21} = constants defined in Eq. (22)
b_1,b_2 = damping constants of platform and rotor dampers, respectively
c_1,c_2 = dimensionless damping constants, $b_1/m\omega_z$ and $b_2/I_s\omega_z$, respectively
C_0 = moment of inertia of undeformed configuration about $0'z'$
C_R = moment of inertia of rotor (including sphere) about $0'z'$
C_p = moment of inertia of platform about $0'z'$
$c\dot{\gamma}$ = friction torque in the motor bearing assembly
I_s = moment of inertia of rotor damper about its center of mass
I = dimensionless inertia, I_s/A_0
k_1,k_2 = dimensionless spring constants defined after Eq. (12)
K_1,K_2 = spring constants of platform and rotor dampers, respectively
m = mass of platform damper
0 = instantaneous mass center of configuration
0′ = mass center of undeformed configuration
$0xyz$ = reference frame associated with deformed configuration
$0'x'y'z'$ = reference frame associated with undeformed configuration
Q = dimensionless parameter defined after Eq. (12)
R = dimensionless inertia, ma^2/A_0
$T_m(t)$ = torque supplied by the motor
t = time
W_1,W_2 = dimensionless variables, ω_x/ω_z and ω_y/ω_z, respectively
x = vector $\{A,B\}$
y = vector $\{\xi,\beta,\xi',\beta'\}$
α = dimensionless rotation rate, $\dot{\gamma}/\omega_z$
β = angular deflection of rotor damper
γ = rotation angle of rotor with respect to platform
Δ = $(C_0 - A_0)/A_0$
ϵ = small parameter
ξ = dimensionless displacement, χ/a
ϕ_1,ϕ_2 = phase lag angles of platform and rotor dampers, respectively
τ = dimensionless time, $\omega_z t$
μ = ratio of m to satellite mass
$\omega_x,\omega_y,\omega_z$ = component absolute angular velocities of $0xyz$ reference frame
χ = linear deflection of platform damper

Superscripts

. = denotes differentiation with respect to time
′ = denotes differentiation with respect to dimensionless time, τ

Received July 10, 1970; revision received November 6, 1970.
* Research Scientist. Member AIAA.

Introduction

THE mechanics of dual-spin satellites has received a great deal of attention in the recent literature because of the immediate application to spin-stabilized communications satellites.[1–8] Of particular interest and study has been the stability of motion in free space conditions of a satellite composed of a "platform" and a "rotor," each rotating at different speeds about a common symmetry axis, and each with its own independent damping.[1–6]

Stability may be assessed adequately by Routh-Hurwitz methods when only one body has a damper.[1] Inclusion of two dampers, one on each body, leads to linearized equations with periodic coefficients. These equations have been investigated by Floquet analysis,[2] but the method does not appear to be well suited for obtaining a concise statement of stability criteria owing to the large number of parameters involved. Also, difficulties arise in obtaining accurate results when the difference in spin rates between the platform and rotor is large (the case which is encountered in practice). Qualitative stability considerations for the problem with two dampers have been proposed as a result of extrapolating Routh-Hurwitz and Floquet results,[1,2] but the arguments put forth do not lead to explicit stability criteria. The energy sink method has been applied to the problem with two dampers, and resulting stability criteria are expressed in terms of energy dissipation rates.[1,6] Stability criteria also have been obtained using function space methods and physical insight of dynamical motions.[3]

In the present work, the formal "method of averaging,"[9–11] is applied to linearized equations describing a satellite with two separate dampers, to obtain a closed-form "first approximation" solution, which is compared with numerical solutions. Stability criteria are obtained and compared with those of prior[1–6] work.

Equations of Motion

Consider a dual-spin satellite composed of a platform which contains a pendulum-type damper and a rotor which contains an internal damper, as shown in Fig. 1. Axes $(0'x'y'z')$ are assigned to the body so that when the damper springs are in their unstretched state, the axis $0'z'$ is a common principal axis of the two bodies (the nominal axis of rotation). The point 0′ coincides with the mass center of the composite body, and the axis $0'x'$ and $0'y'$ are fixed principal axes of the platform. In this study, it is assumed that both rotor and platform are symmetric about the $0'z'$ axis. The rotor rotates with respect to the platform about the $0'z'$ axis by an angle γ, and a rotation rate is maintained by supplying a torque with an internal motor. The platform damper is in static equilibrium with its mass on the $0'z'$ axis, is located a distance "a" from the mass center 0′, and is constrained to oscillate in the $0'x'$ direction. The rotor damper consists of a sphere in a cavity located at 0′, and is constrained to oscillate about an axis transverse to the rotor as is shown in Fig. 1. An additional set of axes ($0xyz$) are assigned to be parallel to the ($0'x'y'z'$) body-fixed axes, so that 0 coincides with the instantaneous mass center of the configuration as the platform damper oscillates.

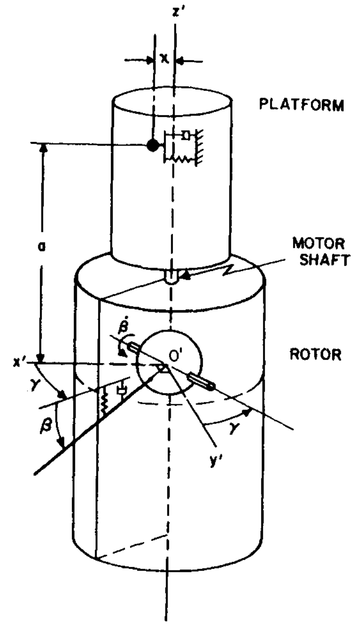

Fig. 1 Dual-spin satellite configuration.

Equations of motion for this satellite may be derived by application of conservation of momentum principles[1] or by variational principles. Since the development of equations of motion for similar configurations is reported in the current literature,[1,8] the derivation will be bypassed, and the equations of interest will be quoted directly.

The equilibrium solution of interest is that corresponding to pure rotation about the $0'z'$ axis where,

$$\omega_x = \omega_y = \chi = \beta = 0 \tag{1}$$

Accordingly, the motion equations may be linearized in ω_x, ω_y, χ, β, and stability then assessed.

Three linear equations arising from conservation of angular momentum are,

$$A_0\dot{\omega}_x - ma\dot{\chi}\omega_z - ma\chi\dot{\omega}_z - I_s\ddot{\beta}\sin\gamma - I_s\dot{\beta}\dot{\gamma}\cos\gamma + (C_0 - A_0)\omega_y\omega_z + C_R\dot{\gamma}\omega_y - (I_s\dot{\beta}\cos\gamma + ma\dot{\chi})\omega_z = 0 \tag{2}$$

$$A_0\dot{\omega}_y + I_s\ddot{\beta}\cos\gamma - I_s\dot{\beta}\dot{\gamma}\sin\gamma + ma\ddot{\chi} - (C_0 - A_0)\omega_x\omega_z - ma\omega_z^2\chi - I_s\dot{\beta}\omega_z\sin\gamma - C_R\dot{\gamma}\omega_x = 0 \tag{3}$$

$$C_0\dot{\omega}_z + C_R\ddot{\gamma} = 0 \tag{4}$$

An equation of motion for the rotor is,

$$C_R(\dot{\omega}_z + \ddot{\gamma}) = -c\dot{\gamma} + T_m(t) \tag{5}$$

where $c\dot{\gamma}$ is a friction torque in the motor bearing assembly, and $T_m(t)$ is a torque supplied by the motor and is designed to maintain $\dot{\gamma}$ as a constant in this instance.

Equations of motion for the platform and rotor dampers are,

$$m(1 - \mu)\ddot{\chi} + ma\dot{\omega}_y + ma\omega_z\omega_x + b_1\dot{\chi} + [K_1 - m\omega_z^2(1 - \mu)]\chi = 0 \tag{6}$$

$$I_s\ddot{\beta} - I_s\dot{\gamma}\cos\gamma\omega_x - I_s\sin\gamma\dot{\omega}_x - I_s\dot{\gamma}\sin\gamma\omega_y + I_s\cos\gamma\dot{\omega}_y + b_2\dot{\beta} + K_2\beta = 0 \tag{7}$$

Equations (4) and (5) possess a "steady-state" solution,

$$\omega_z = \text{constant} \qquad \gamma = \dot{\gamma}_0 t \tag{8}$$

(where $\dot{\gamma}_0$ is a constant), when the torque $T_m(t)$ is designed to overcome friction and to damp the relative oscillations between platform and rotor, as is the usual engineering practice (for example, $T_m(t) = c\dot{\gamma}_0 + T_1\ddot{\gamma}$, where T_1 is a constant, will achieve the required result).

Equation (8) may be substituted into Eqs. (2, 3, 6, and 7), and the result expressed in the dimensionless form,

$$W_x' + QW_y - 2R\xi' - I\sin\gamma\beta'' - I(\alpha + 1)\cos\gamma\beta' = 0 \tag{9}$$

$$W_y' - QW_x + R(\xi'' - \xi') + I\cos\gamma\beta'' - I(\alpha + 1)\sin\gamma\beta' = 0 \tag{10}$$

$$(1 - \mu)\xi'' + W_y' + W_x + c_1\xi' + k_1\xi = 0 \tag{11}$$

$$\beta'' + c_2\beta' + k_2\beta - \alpha\cos\gamma W_x - \sin\gamma W_x' + \cos\gamma W_y' - \alpha\sin\gamma W_y = 0 \tag{12}$$

where $\tau = \omega_z t$, the primes denote differentiation with respect to τ, and

$$\alpha = \dot{\gamma}_0/\omega_z, \qquad \xi = \chi/a$$

$$W_x = \omega_x/\omega_z, \qquad W_y = \omega_y/\omega_z$$

$$Q = \{(C_0 - A_0)/A_0\} + C_R\alpha/A_0$$

$$I = I_s/A_0, \qquad R = ma^2/A_0$$

$$c_1 = b_1/m\omega_z, \qquad c_2 = b_2/I_s\omega_z$$

$$k_1 = \{K_1 - (1 - \mu)\}/m\omega_z^2, \qquad k_2 = K_2/I_s\omega_z^2$$

$$\gamma = \dot{\gamma}_0 t = \alpha\tau$$

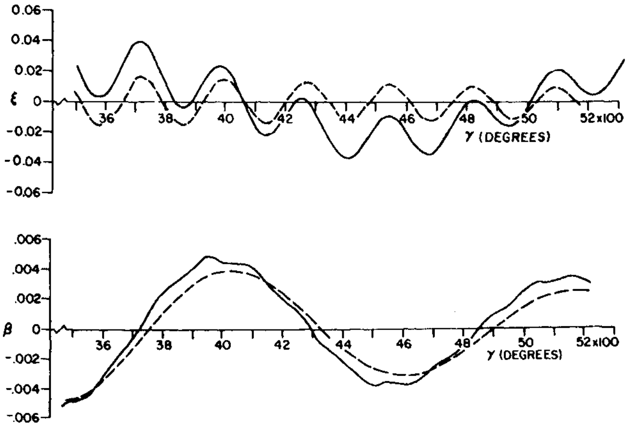

Fig. 2 A, B, ξ, and β vs γ.

Note that $\alpha > 0$ by definition; i.e., if $\alpha < 0$, one must invert the definition of "platform" and "rotor."

Transformation to Standard Form

The parameters R and I are very small in actual practice. Upon recognizing this, Eqs. (9–12) may be transformed into a form suitable for application of an asymptotic method of solution. Introducing new variables $A(\tau)$ and $B(\tau)$ by the transformation,

$$W_x = A(\tau)\cos Q\tau + B(\tau)\sin Q\tau \qquad (13a)$$

$$W_y = A(\tau)\sin Q\tau - B(\tau)\cos Q\tau \qquad (13b)$$

and substituting Eqs. (13) into Eqs. (9 and 10) leads to,

$$A' = \epsilon\{\Lambda_1 \cos Q\tau + \Lambda_2 \sin Q\tau\} \qquad (14a)$$

$$B' = \epsilon\{\Lambda_1 \sin Q\tau - \Lambda_2 \cos Q\tau\} \qquad (14b)$$

where ϵ is a small parameter (artificially introduced as shown below), Λ_1 and Λ_2 are given by,

$$\Lambda_1 = \{2R\xi' + I\sin\gamma\beta'' + I(\alpha + 1)\cos\gamma\beta'\}/\epsilon$$

$$\Lambda_2 = \{R(\xi - \xi'') - I\cos\gamma\beta'' + I(\alpha + 1)\sin\gamma\beta'\}/\epsilon$$

and R/ϵ, I/ϵ, are of order of magnitude k_1, k_2, etc.

Substitution of Eq. (13) into Eqs. (11) and (12) leads to,

$$(1 - \mu)\xi'' + c_1\xi' + k_1\xi + A(Q + 1)\cos Q\tau + B(Q + 1)\sin Q\tau + \epsilon\Lambda_2 = 0 \qquad (15a)$$

$$\beta'' + c_2\beta' + k_2\beta + A(Q - \alpha)\cos(Q - \alpha)\tau + B(Q - \alpha)\sin(Q - \alpha)\tau + \epsilon(\Lambda_2\cos\gamma - \Lambda_1\sin\gamma) = 0 \qquad (15b)$$

At this point it becomes evident that Eqs. (14) and (15) may be rearranged into the form,

$$x' = \epsilon X(x,y) \qquad y' = Y(x,y,\epsilon) \qquad (16)$$

where x and y are 2 and 4 dimensional vectors: $x = \{A,B\}$, $y = \{\xi,\beta,\xi',\beta'\}$. Solutions of Eqs. (16) have been found and established by the formal method of averaging.[9–11] Solutions valid to any degree of approximation may be obtained. Briefly, one seeks a solution of the form,

$$x = \bar{x} + \sum_{k=1}^{\infty} \epsilon^k u^k \qquad y = \bar{y} + \sum_{k=1}^{\infty} \epsilon^k v^k \qquad (17)$$

where $\bar{x}$ and $\bar{y}$ are the "averaged" solutions, and u^k and v^k are time-varying functions. The functions $\bar{x},\bar{y},u^k$ and v^k are obtained by solving differential equations constructed by formal procedures outlined in Ref. 9, and are usually easier to solve than the original Eq. (16). To obtain a solution valid to a "first approximation," (i.e., $x = \bar{x}$, $y = \bar{y}$), one first solves Eq. (16) with $\epsilon = 0$, to obtain a solution

$$x = \text{const} = \bar{x} \qquad y = \zeta(x,\tau) \qquad (18)$$

and then constructs the equations for $\bar{x}$ and $\bar{y}$,

$$\frac{d\bar{x}}{d\tau} = \lim_{T\to\infty} \frac{1}{T}\int_{\tau}^{\tau+T} \epsilon X[\bar{x},\zeta(\bar{x},t)]dt \qquad (19a)$$

$$y = \zeta(x,\tau) \qquad (19b)$$

"First Approximation" Solution

The method as outlined will be applied directly to Eqs. (14) and (15) [without actually transforming to the form of Eq. (16)], to obtain a "first approximation" solution. The equations for the solution corresponding to Eq. (18) are,

$$A' = 0 \qquad B' = 0$$

together with Eqs. (15). Upon solving these, the "steady-state" values are found to be,

$$\bar{\xi} = -\frac{(Q + 1)[A\cos(Q\tau - \phi_1) + B\sin(Q\tau - \phi_1)]}{[(k_1 - Q^2)^2 + (c_1Q)^2]^{1/2}} \qquad (20a)$$

$$\bar{\beta} = -\frac{(Q - \alpha)[A\cos\{(Q - \alpha)\tau - \phi_2\} + B\sin\{(Q - \alpha)\tau - \phi_2\}]}{[\{k_2 - (Q - \alpha)^2\}^2 + \{c_2(Q - \alpha)\}^2]^{1/2}} \qquad (20b)$$

where

$$\tan\phi_1 = c_1Q/(k_1 - Q^2)$$
$$\tan\phi_2 = c_2(Q - \alpha)/[k_2 - (Q - \alpha)^2] \qquad (20c)$$

and $1 - \mu \simeq 1$. The above expressions for ϕ_1 and ϕ_2 are multivalued, and it is important to be consistent when choosing the branch in numerical calculations.

Substitution of Eqs. (20) into (14) and "averaging" as in Eq. (19a) results in (after lengthy but straightforward calculation) the following differential equations:

$$(d\bar{A}/d\tau) = a_{11}\bar{A} - a_{21}\bar{B}, \qquad (d\bar{B}/d\tau) = a_{21}\bar{A} + a_{11}\bar{B} \qquad (21)$$

where

$$a_{11} = -\frac{c_1RQ(1 + Q)^3}{2[(k_1 - Q^2)^2 + (c_1Q)^2]} - \frac{c_2I(1 + Q)(Q - \alpha)^3}{2[\{k_2 - (Q - \alpha)^2\}^2 + \{c_2(Q - \alpha)\}^2]} \qquad (22a)$$

$$a_{21} = \frac{R(1 + Q)^3(k_1 - Q^2)}{2[(k_1 - Q^2)^2 + (c_1Q)^2]} + \frac{I(1 + Q)(Q - \alpha)^2\{k_2 - (Q - \alpha)^2\}}{2[\{k_2 - (Q - \alpha)^2\}^2 + \{c_2(Q - \alpha)\}^2]} \qquad (22b)$$

[The identity $\sin\phi_1 = c_1Q/\{(k_1 - Q^2)^2 + (c_1Q)^2\}^{1/2}$, and similar ones in $\cos\phi_1$, $\sin\phi_2$, and $\cos\phi_2$, have been utilized in deriving Eq. (22).]

Eqs. (21) are easily solved to obtain,

$$\bar{A}(\tau) = e^{a_{11}\tau}\{A_0\cos a_{21}\tau - B_0\sin a_{21}\tau\} \qquad (23a)$$

$$\bar{B}(\tau) = e^{a_{11}\tau}\{A_0\sin a_{21}\tau + B_0\cos a_{21}\tau\} \qquad (23b)$$

where A_0 and B_0 are initial values. Thus, in accordance with the previous discussion a first approximation solution is,

$$A = \bar{A},\ B = \bar{B},\ \xi = \bar{\xi},\ \beta = \bar{\beta} \qquad (24)$$

where $\bar{A},\bar{B}$, and $\bar{\xi},\bar{\beta}$ are as given in Eqs. (23) and (20).

Thus Eq. (24) is a solution to Eqs. (14) and (15), and after transformation by Eq. (13), also a solution to their exact equivalent, namely Eqs. (9–12). The solution may be expected to be approximately valid whenever the right hand side of Eq. (14) is "small," which is true when R and I are sufficiently small. Inaccuracy may arise when the dampers are excited at "near resonance" conditions, in which case χ and β [and consequently the right-hand side of Eq. (14)] may be very large.

The solution accuracy may be improved to any degree desired, at the expense of laborious but straightforward calculation, by invoking the theory for the higher order calculation as outlined in Refs. 9–11.

Eqs. (9–12) have been solved numerically for a number of selected cases, and the solutions have been compared with Eq. (23) after a time interval for which ξ and β had reached "steady state" and the values of α were less than 100. In all cases agreement was found. Figure 2 shows a sample set of comparisons for a rather extreme case where R and I are 0.1. The variables of greatest interest, A and B, agree well. The variables ξ and β agree less well; notably, ξ has a long subharmonic period which does not appear in the approximate solution. Presumably the situation could be improved by calculating a higher order approximation. In test checks at the very small values of R and I encountered in practice, ξ, β, A, B agreements were very close. However, A and B vary much more slowly, and the results did not lend themselves well to illustrative graphical comparisons.

Stability

The analytical stability criterion deduced from Eq. (23) is,

$$a_{11} \leqslant 0 \text{ implies stability; } a_{11} > 0 \text{ implies instability} \quad (25)$$

where a_{11} is given by Eq. (22a). This criterion has been checked further for a number of critical cases against results obtained by numerical application of Floquet analysis (details of the numerical method used are described in, e.g., Ref. 3). Results presented in Table 1 show that complete agreement was found. In computing the Floquet exponents for cases when α was large, ($\alpha > 1000$) and R and I were very small (which is the case of interest in current satellite design), computational inaccuracies were encountered, and stability assessments were often questionable. The results listed in Table 1 exclude consideration of such cases.

The criterion (25) is similar in form to that obtained by Puri and Gido[3] (for a different configuration) by the "damper-reaction torque and quadratic function method." With some manipulation, the criterion may also be cast into a form equivalent to that obtained by the energy sink analysis of Refs. 1 or 6. This is not entirely unexpected, since both the energy sink and damper-reaction torque methods are physically motivated versions of an averaging technique (a "first approximation" level of averaging evidently). In contrast, the development offered herein is more precise, is based on established theory, and will yield a complete solution of the governing equations to any degree of approximation desired. The numerical work reported in the preceding section indicates that the first-approximation accuracy is adequate for stability assessment of flight cases of interest but may be lacking in describing the actual solution behavior.

When the rotor damper is absent, the problem of this paper reduces to the one treated in Ref. 1 by the exact Routh-Hurwitz method. When $I = 0$, criterion (25), obtained by the method of averaging, reduces to, $Q = \Delta + J\alpha > 0$, since $(1 + Q)$ is necessarily greater than zero. This statement corresponds exactly to the criterion (42) of Ref. 1, since the Q of this paper is equal to (λ/ω_A) of Ref. 1. Hence the two sets of results agree. Expression (25) also agrees with the useful qualitative picture of destabilizing and stabilizing effects put forth in Refs. 1 and 2.

Factors Influencing the Design of Satellites

Upon recalling the definition of Q, one sees that $(1 + Q) > 0$ for all physically realizable configurations. Also, α, c_1, c_2, and the denominators of Eq. (22a) are positive. Upon considering these facts in conjunction with Eq. (22a), one may easily show that: a) the platform damper exerts a stabilizing influence if and only if $Q > 0$, i.e., when

$$\{(C_0/A_0) - 1\} + (C_R/A_0)\alpha > 0 \quad (26a)$$

and b) the rotor damper exerts a stabilizing influence if and only if $(Q - \alpha) > 0$, i.e., when

$$\{(C_R/A_0) - 1\}(1 + \alpha) + (C_P/A_0) > 0 \quad (26b)$$

where

$$C_0 = C_R + C_P$$

Let us consider briefly the case when α is large, say 100,000, and $C_R/A_0 > 0.001$, which is the case of interest in the design of current communications satellites. Since C_0/A_0, C_R/A_0, and C_P/A_0 are limited to [0,2], one sees that the platform damper is always stabilizing, but the rotor damper is stabilizing if and only if $C_R/A_0 > 1$.

By examination of Eq. (22a) it is seen that the stabilizing or destabilizing effect is greatly magnified for a given damper when it is excited at or near resonance conditions. Hence it appears that in practical satellite design for which the rotor damping is of the destabilizing variety (such as in the gyrostat design, where $C_R/A_0 < 1$) it is essential to be able to detect and describe accurately all sources of damping.

Table 1 Comparison of analytical and Floquet stability calculations (for all calculations, $c_1 = c_2 = 0.1$, $R = 0.1$)

I	Δ	J	α	k_1	k_2	Remarks	Analytical or Floquet criterion
0.1	0.2	1.1	1	2	2	$\phi_1 > 0$ $\phi_2 > 0$	stable
0.1	0.5	0.4	1	4	4	$\phi_1 < 0$ $\phi_2 < 0$	unstable
0.1	−0.5	0.4	5	5	5	$\phi_1 > 0$ $\phi_2 < 0$	unstable
0.1	−0.5	0.4	5	4	13	$\phi_1 > 0$ $\phi_2 < 0$	unstable
0	−0.5	0.25	10	2	100		stable
0	−0.5	0.25	1	2	100		unstable

Concluding Remarks

The analytical solution and stability criterion obtained by the formal method of averaging, have been verified by comparison with numerically computed solutions over a range of parameter values for which the numerical solutions are accurate. When the rotor spin rate is large relative to the platform and the ratio of damper mass to total mass is very small, inaccuracies arise in computation of Floquet exponents and jeopardize the validity of stability results obtained by that method. In contrast, the method of averaging solution developed herein approaches the true solution with increasing accuracy as $(1/\alpha)$, R, and I, tend to small values. Hence, the method of averaging appears to be well suited to this problem.

References

[1] Likins, P. W., "Attitude Stability Criteria for Dual Spin Spacecraft," *Journal of Spacecraft and Rockets*, Vol. 4, No. 12, Dec. 1967, pp. 1638–1643.

[2] Mingori, D. L., "Effects of Energy Dissipation on the Attitude Stability of Dual Spin Satellites," *AIAA Journal*, Vol. 7, No. 1, Jan. 1969, pp. 20–27.

[3] Puri, N. N. and Gido, J. F., "Nutational Stability Criteria for a Dual Spin Spacecraft—The Damper Reaction Torque and Quadratic Function Method," Paper AT103, 19th Congress of the International Astronautical Federation, New York, 1968.

[4] Likins, P. W., Tseng, G. T., and Mingori, D. L., "Stable Limit Cycles due to Nonlinear Damping in Dual-Spin Spacecraft," AIAA Paper 70-1044, Santa Barbara, Calif., 1970.

[5] Pringle, R., Jr., "The Stability of Force-Free Motions of a Dual-Spin Spacecraft," *AIAA Journal*, Vol. 7, No. 6, June 1969, pp. 1054–1063.

[6] Papis, T., "Analysis of the Attitude Stability of a Dual Spin Spacecraft by the Energy Method," AIAA Paper 70-973, Santa Barbara, Calif., 1970.

[7] Roberson, R. E., "Equilibria of Orbiting Gyrostats," *The Journal of the Astronautical Sciences*, Vol. 15, No. 5, Sept.–Oct. 1968, pp. 242–248.

[8] Gale, A. H. and Likins, P. W., "Influence of Flexible Appendages on Dual-Spin Spacecraft Dynamics and Control," *Journal of Spacecraft and Rockets*, Vol. 7, No. 9, Sept. 1970, pp. 1049–1056.

[9] Volosov, V. M., "Averaging in Systems of Ordinary Differential Equations," *Russian Math Surveys*, Vol. 17, 1962, pp. 1–126.

[10] Volosov, V. M., "The Method of Averaging," *Soviet Mathematics-Doklady*, Vol. 2, 1961, pp. 221–224.

[11] Volosov, V. M., "Higher Approximations in Averaging," *Soviet Mathematics-Doklady*, Vol. 2, 1961, pp. 382–385.